Globally Optimal Gasketed Plate Heat Exchanger Design Optimization

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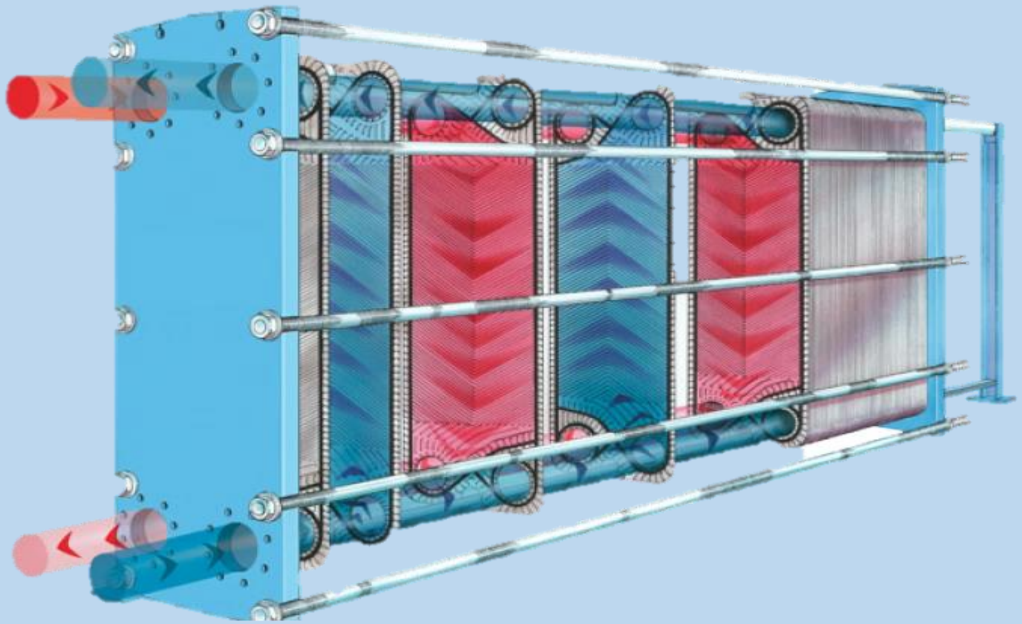
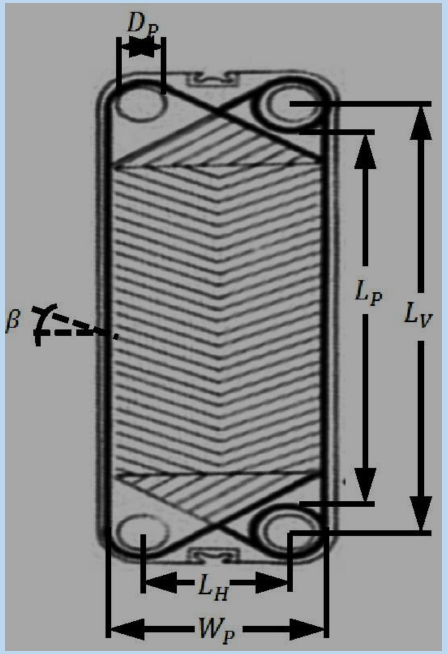
1) Introduction





2) Thermo-fluidodynamic behavior

$$Nu = C\ Re^{a} Pr^{0,33} \left({}^{\mu}/_{\mu_w}\right)^{0,17}$$

$$\frac{\Delta P_c}{\rho g} = \frac{f\ Np\ v^2 \left({}^{\mu}/_{\mu_w}\right)^{-0,17}}{Dh\ 2\ g} \qquad f = K Re^{-z}$$

$$\frac{\Delta P_p}{\rho g} = 1{,}4 Np \frac{v^2}{2\ g}$$

β	*Re*	*C*	*a*
≤30°	≤10	0,718	0,349
	>10	0,348	0,663
45°	≤10	0,718	0,349
	10 - 100	0,400	0,598
	>100	0,300	0,663
50°	< 20	0,630	0,333
	20 – 300	0,291	0,591
	> 300	0,130	0,732
60°	< 20	0,562	0,326
	20 – 400	0,306	0,529
	> 400	0,108	0,703
≥ 65°	< 20	0,562	0,326
	20 – 500	0,331	0,503
	>500	0,087	0,718

Re	*K*	*z*
≤10	200	1
10-100	77,60	0,589
>100	11,96	0,183
≤15	188	1
15 – 300	73,16	0,652
>300	5,764	0,206
< 20	136	1
20 – 300	45	0,631
>300	3,088	0,161
< 40	96	1
40 – 400	12,96	0,457
> 400	3,04	0,215
< 50	96	1
50 – 500	11,2	0,451
>500	2,556	0,213

3) LMTD method

Objective function

min A = $\hat{\Phi} N_t L_w L_p$

Restrictions

$\Delta P \leq \Delta P_{max}$

$v_{min} \leq v \leq v_{max}$

$$\mathrm{A} \geq \left(1 + \frac{Aex}{100}\right) Areq$$

Discrete design variables

- Number of plates (170 different possibilities)
- Plate size (5 different plates)
- Corrugation angle
- Number of passes of each stream (1-1;1-2;2-1;2-2)

4) Optimization approaches

Mixed-integer non linear programming (MINLP)

Global solvers

- Baron
- Antigone

Local solvers

- Sbb
- Dicopt

Mixed-integer linear programming (MILP)

- Linearization techniques
- Alternative representation method

Set-trimming

4) Optimization approaches

Mixed-integer non linear programming (MINLP)

Seven binary variables:

yNt(snt) Number of plates
yP(sp) Plate size
yNph(snph) Hot stream number of passes
yNpc(snpc) Cold stream number of passes
yβ(sβ) Corrugation angle
yRh(sreh) Hot stream reynolds range
yRc(srec) Cold stream Reynolds range

$$Nt = \sum_{snt} pNt(snt) yNt(snt); \quad \sum_{snt} yNt(snt) = 1$$

$$Lw = \sum_{sp} pLw(sp) yP(sp); \quad \sum_{sp} yP(p) = 1$$

$$Lp = \sum_{sp} pLp(sp) yP(sp); \quad \sum_{sp} yP(p) = 1$$

$$Np = \sum_{snt} pNp\, ynp(snp); \quad \sum_{snp} yNp(snp) = 1$$

4) Optimization approaches

Mixed-integer linear programming (MILP): linearization technique

Seven binary variables and eight positive variables:

$wpNt_{snt,sp}$

$wNphNtP_{snt,sp,snph}$

$wNpcNtP_{snt,sp,snpc}$

$wNphNtPRh\beta_{snph,snt,sp,sreh,s\beta}$

$wNpcNtPRc\beta_{snpc,snt,sp,srec,s\beta}$

$wNphNpcNtP_{snph,snpc,snt,sp}$

$wNphP_{snph,sp}$

$wNpcP_{snpc,sp}$

$$A = \hat{\Phi} \sum_{snt} \sum_{sp} \widehat{pNt}_{snt} \widehat{pLp}_{sp} \widehat{pLw}_{sp} yP_{sp} yNt_{snt}$$

$$wpNt_{snt,sp} \leq yNt_{snt}$$

$$wpNt_{snt,sp} \leq yP_{sp}$$

$$wpNt_{snt,sp} \geq yP_{sp} + yNt_{snt} - 1$$

$$0 \leq wpNt_{snt,sp} \leq 1$$

4) Optimization approaches

Mixed-integer linear programming (MILP): linearized equations

Mass flux

$$Gh = \frac{2\,\widehat{mh}}{\hat{b}} \sum_{snph} \sum_{sp} \sum_{snt} \frac{\widehat{pNph}_{snph}}{\widehat{pLw}_{sp}(\widehat{pNt}_{snt} - 1)} wNphNtP_{snt,sp,snph}$$

$$Gc = \frac{2\,\widehat{mc}}{\hat{b}} \sum_{snpc} \sum_{sp} \sum_{snt} \frac{\widehat{pNpc}_{snpc}}{\widehat{pLw}_{sp}(\widehat{pNt}_{snt} - 1)} wNpcNtP_{snt,sp,snpc}$$

Heat transfer rate equation

$$\left[\sum_{s\beta}\sum_{sreh}\sum_{snph}\sum_{sp}\sum_{snt} \frac{\widehat{kh}\,\widehat{pCh}_{s\beta,sreh}\widehat{Pr}_h^{0,33}}{\widehat{Dhyd}} \left(\frac{\widehat{2\,\dot{m}_h}\widehat{Dhyd}}{\hat{b}\,\widehat{mih}} \frac{\widehat{pNph}_{snph}}{\widehat{pLw}_{sp}(\widehat{pNt}_{snt}-1)}\right)^{\widehat{pAh_{s\beta,sreh}}}\right]^{-1} wNphNtPRh\beta_{snph,snt,sp,sreh,s\beta}$$
$$+\left[\sum_{s\beta}\sum_{srec}\sum_{snpc}\sum_{sp}\sum_{snt} \frac{\widehat{kc}\,\widehat{pCc}_{s\beta,srec}\widehat{Pr}_c^{0,33}}{\widehat{Dhyd}} \left(\frac{\widehat{2\,\dot{m}_c}\widehat{Dhyd}}{\hat{b}\,\widehat{mic}} \frac{\widehat{pNpc}_{snpc}}{\widehat{pLw}_{sp}(\widehat{pNt}_{snt}-1)}\right)^{\widehat{pAc_{s\beta,srec}}}\right]^{-1} wNpcNtPRc\beta_{snpc,snt,sp,srec,s\beta} + \frac{\hat{t}}{\widehat{k_w}} + \widehat{Rfh} + \widehat{Rfc}$$
$$\leq \sum_{snph}\sum_{snpc}\sum_{snt}\sum_{sp} \frac{\widehat{phi}\,\widehat{\Delta T_{Lm}}\,(\widehat{pNt}_{snt}-2)\widehat{pLw}_{sp}\widehat{pPf}_{snph,snpc}\widehat{pLp}_{sp}}{\widehat{Q_r}\left(1+\frac{\widehat{E_{xc}}}{100}\right)} wNphNpcNtP_{snph,snpc,snt,sp}$$

4) Optimization approaches

Mixed-integer linear programming (MILP): linearized equations

Pressure drop

$$\frac{1{,}4\,\widehat{roh}}{2}\left(\frac{4\,\widehat{mh}}{\widehat{roh}\,\pi}\right)^2 \sum_{snph}\sum_{sp}\frac{\widehat{pNph}_{snph}}{\widehat{pDp}_{sp}{}^4} wNphP_{snph,sp} + \sum_{s\beta}\sum_{sreh}\sum_{snph}\sum_{sp}\sum_{snt}\frac{\widehat{pNph}_{snph}\left(\widehat{pLp}_{sp}+\widehat{pDp}_{sp}\right)\widehat{pKc}_{s\beta,sreh}}{2\widehat{Dhyd}\,\widehat{roh}}\left(\frac{2\widehat{mh}\,\widehat{pNph}_{snph}}{\widehat{bp}\,\widehat{pLw}_{sp}\left(\widehat{pNt}_{snt}-1\right)}\right)^2\left(\frac{2\,\widehat{Dhyd}\,\widehat{mh}}{\hat{b}\,h}\frac{\widehat{pNph}_{snph}}{\widehat{pLw}_{sp}\left(\widehat{pNt}_{snt}-1\right)}\right)^{-\widehat{pNc}_{s\beta,sreh}} wNphNtPRh\beta_{snph,snt,sp,sreh,s\beta} \le \Delta Phmax$$

$$\frac{1{,}4\,\widehat{roc}}{2}\left(\frac{4\,\widehat{mc}}{\widehat{roc}\,\pi}\right)^2 \sum_{snpc}\sum_{sp}\frac{\widehat{pNpc}_{snpc}}{\widehat{pDp}_{sp}{}^4} wNpcP_{snpc,sp} + \sum_{s\beta}\sum_{srec}\sum_{snpc}\sum_{sp}\sum_{snt}\frac{\widehat{pNpc}_{snpc}\left(\widehat{pLp}_{sp}+\widehat{pDp}_{sp}\right)\widehat{pKc}_{s\beta,srec}}{2\widehat{Dhyd}\,\widehat{roc}}\left(\frac{2\widehat{mc}\,\widehat{pNpc}_{snpc}}{\widehat{bp}\,\widehat{pLw}_{sp}\left(\widehat{pNt}_{snt}-1\right)}\right)^2\left(\frac{2\,\widehat{Dhyd}\,\widehat{mc}}{\hat{b}\,\widehat{mic}}\frac{\widehat{pNpc}_{snpc}}{\widehat{pLw}_{sp}\left(\widehat{pNt}_{snt}-1\right)}\right)^{-\widehat{pNc}_{s\beta,srec}} wNpcNtPRc\beta_{snpc,snt,sp,srec,s\beta} \le \Delta Pcmax$$

Objective function

$$A = \hat{\Phi}\sum_{snt}\sum_{sp}\widehat{pNt}_{snt}\widehat{pLp}_{sp}\widehat{pLw}_{sp}wpNt_{snt,sp}$$

4) Optimization approaches

Mixed-integer linear programming (MILP): linearized equations

Constrains of corrugation parameter

$$\frac{2\ \widehat{Dhyd}\ \widehat{mh}}{\hat{b}\ \widehat{mih}}\sum_{snph}\sum_{sp}\sum_{snt}\frac{\widehat{pNph}_{snph}}{\widehat{pLw}_{sp}(\widehat{pNt}_{snt}-1)}wNphNtP_{snt,sp,snph}-\sum_{sreh=1}^{2}\widehat{gammah}_{s\beta,sreh+1}yRh_{sreh}-\widehat{Redh}_{max}yRh_{sreh=3}\leq\ \widehat{UB}(1-y\beta_{s\beta})$$

$$\frac{2\ \widehat{Dhyd}\ \widehat{mh}}{\hat{b}\ \widehat{mih}}\sum_{snph}\sum_{sp}\sum_{snt}\frac{\widehat{pNph}_{snph}}{\widehat{pLw}_{sp}(\widehat{pNt}_{snt}-1)}wNphNtP_{snt,sp,snph}-\sum_{sreh=1}^{3}\widehat{gammah}_{s\beta,sreh}yRh_{sreh}\geq\ \widehat{LB}(1-y\beta_{s\beta})$$

$$\frac{2\ \widehat{Dhyd}\ \widehat{mc}}{\hat{b}\ \widehat{mic}}\sum_{snpc}\sum_{sp}\sum_{snt}\frac{\widehat{pNpc}_{snpc}}{\widehat{pLw}_{sp}(\widehat{pNt}_{snt}-1)}wNpcNtP_{snt,sp,snpc}-\sum_{srec=1}^{2}\widehat{gammac}_{s\beta,srec+1}yRc_{srec}-\widehat{Redc}_{max}yRc_{srec=3}\leq\ \widehat{UB}(1-y\beta_{s\beta})$$

$$\frac{2\ \widehat{Dhyd}\ \widehat{mc}}{\hat{b}\ \widehat{mic}}\sum_{snpc}\sum_{sp}\sum_{snt}\frac{\widehat{pNpc}_{snpc}}{\widehat{pLw}_{sp}(\widehat{pNt}_{snt}-1)}wNpcNtP_{snt,sp,snpc}-\sum_{srec=1}^{3}\widehat{gammac}_{s\beta,srec}yRc_{srec}\geq\ \widehat{LB}(1-y\beta_{s\beta})$$

4) Optimization approaches

Mixed-integer linear programming (MILP): Alternative approach

Three binary variables:

$yrow_{snt,sp,snph,snpc,sa}$

$yreh_{sreh}$

$yrec_{srec}$

two positive variables(‘w’)

$wNphNpcNtPRhA_{(snph,snpc,snt,sp,sreh,sa)}$

$wNphNpcNtPRcA_{(snph,snpc,snt,sp,srec,sa)}$

$$\widehat{PN}t_{(snt,sp,snph,snpc,sa)} = \widehat{pN}t_{(snt)}$$

$$Nt_{(snt,sp,snph,snpc,sa)} = \sum_{snt,sp,snph,snpc,sa} \widehat{PN}t_{(snt,sp,snph,snpc,sa)}\, yrow_{snt,sp,snph,snpc,sa}$$

4) Optimization approaches: Set-trimming

Search Space

All combinations of parameters related to the design variables: total number of plates, plate size, Chevron angle, number of passes for the hot and cold streams

$$v_{min} \leq v \leq v_{max}$$

$$\Delta P \leq \Delta P_{max}$$

$$\mathrm{A} \geq \left(1 + \frac{Aex}{100}\right) Areq$$

Feasible Alternatives

Combinations of parameters related to the design variables that satisfy all constraints

Optimal Solution

Pick the alternative with the lowest heat transfer area

4) Optimization approaches: Preliminary results

Example	Results	Set-trimming	yrow	w	Baron	Sbb	Dicopt	Antigone with bounds	Baron with bounds
1	**Area**	399,855	399,855	399,855	399,855	399,855		399,855	399,855
	Execution_Time	0,156	4,14	3,2	0,078	0,031		0,016	0,031
	Elapser_Time	**0,245999**	**35,799**	**536,123**	**27,533**	**2,1829996**		**123,233**	**38,077**
2	**Area**	200,7188	200,719	200,719	200,7188	200,7188		200,7188	200,7188
	Execution_Time	0,093	4,108	1,953	0,063	0,016		0,031	0,032
	Elapser_Time	**0,2389991**	**101,524**	**3414,08**	**18,2749996**	**3,9489998**		**212,669**	**11,048**
3	**Area**	105,4135	105,414	105,414	113,138	120,209		120,55	105,4135
	Execution_Time	0,062	3,813	2,75	0,016	0,016		0,016	0
	Elapser_Time	**0,128**	**106,786**	**805,085**	**356,302**	**0,846999**		**174,862**	**9,854**
4	**Area**	590,4385	590,439	590,439	590,4385	686,067		590,4385	597,51
	Execution_Time	0,047	3,75	2,703	0,016	0,016		0,063	0,016
	Elapser_Time	**0,1199973**	**6,48**	**602,74**	**9,001**	**3,54**		**51,046**	**7,211**
5	**Area**	238,2634	238,263	238,263	239,71	238,2634		244,04	238,2634
	Execution_Time	0,063	3,859	2,625	0,016	0,078		0	0,016
	Elapser_Time	**0,118**	**17,111**	**296,112**	**12,817**	**1,81599**		**11,668**	**11,23**

- All linear approaches have the same results.
- Set-trimming is the faster approach.

5) Discretization method: Wall effects and temperature dependence

Hot stream first pass

$$\frac{dTh}{dx} = \frac{-w}{m_h Cp_h}[Ue(Th - Tc,e) + Ud(Th - Tc,d)]$$

Hot stream second passes

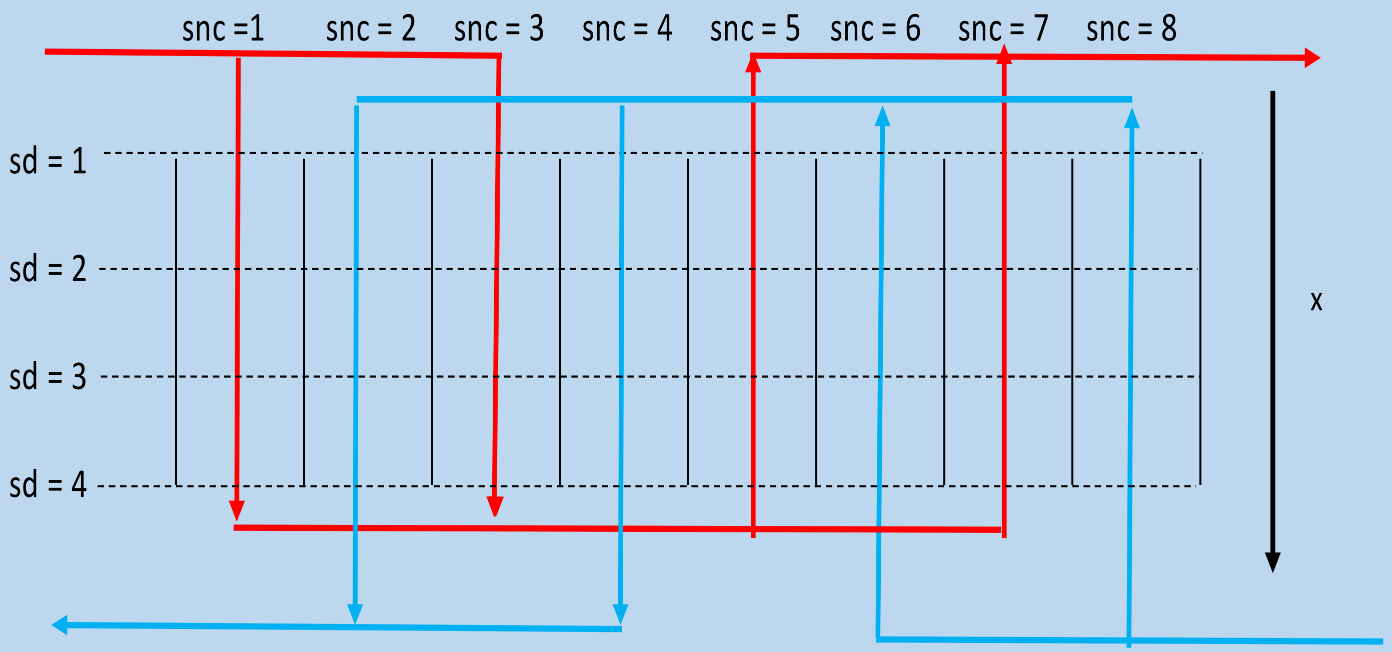
$$\frac{dTh}{dx} = \frac{w}{m_h Cp_h}[Ue(Th - Tc,e) + Ud(Th - Tc,d)]$$

Cold stream first pass

$$\frac{dTc}{dx} = \frac{-w}{m_c Cp_c}[Ue(Th,e - Tc) + Ud(Th,d - Tc)]$$

Cold stream second passes

$$\frac{dTc}{dx} = \frac{w}{m_c Cp_c}[Ue(Th,e - Tc) + Ud(Th,d - Tc)]$$



5) Discretization method

Hot stream first pass: sd = 2 to 4

$$\frac{(T(snc,sd)-T(snc,sd-1))}{dx} = -\frac{Lw}{(VazC(snc)*Cp(snc,sd))}\left\{U(snc,sd)\left[\frac{T(snc,sd)+T(snc,sd-1)}{2}-\frac{T(snc+1,sd)+T(snc+1,sd-1)}{2}\right]+U(snc-1,sd)\left[\frac{T(snc,sd)+T(snc,sd-1)}{2}-\frac{T(snc-1,sd)+T(snc-1,sd-1)}{2}\right]\right\}$$

Hot stream second pass: sd = 1 to 3

$$\frac{(T(snc,sd)-T(snc,sd+1))}{dx} = \frac{Lw}{(VazC(snc)*Cp(snc,sd))}\left\{U(snc,sd)\left[\frac{T(snc,sd+1)+T(snc,sd)}{2}-\frac{T(snc+1,sd+1)+T(snc+1,sd)}{2}\right]+U(snc-1,sd)\left[\frac{T(snc,sd+1)+T(snc,sd)}{2}-\frac{T(snc-1,sd+1)+T(snc-1,sd)}{2}\right]\right\}$$

Cold stream first pass: sd = 1 to 3

$$\frac{(T(snc,sd)-T(snc,sd+1))}{dx} = -\frac{Lw}{(VazC(snc)*Cp(snc,sd))}\left\{U(snc,sd)\left[\frac{T(snc+1,sd+1)+T(snc+1,sd)}{2}-\frac{T(snc,sd+1)+T(snc,sd)}{2}\right]+U(snc-1,sd)\left[\frac{T(snc-1,sd+1)+T(snc-1,sd)}{2}-\frac{T(snc,sd+1)+T(snc,sd)}{2}\right]\right\}$$

Cold stream second pass: sd = 2 to 4

$$\frac{(T(snc,sd)-T(snc,sd-1))}{dx} = \frac{Lw}{(VazC(snc)*Cp(snc,sd))}\left\{U(snc,sd)\left[\frac{T(snc+1,sd)+T(snc+1,sd-1)}{2}-\frac{T(snc,sd)+T(snc,sd-1)}{2}\right]+U(snc-1,sd)\left[\frac{T(snc-1,sd)+T(snc-1,sd-1)}{2}-\frac{T(snc,sd)+T(snc,sd-1)}{2}\right]\right\}$$

5) Discretization method

Next steps

- Add the pressure drop calculation
- Develop a minimization routine